ENGINEERING MECHANICS-STATICS
CHAPTER 1

[STATICS] INTRODUCTION TO MECHANICS

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References:

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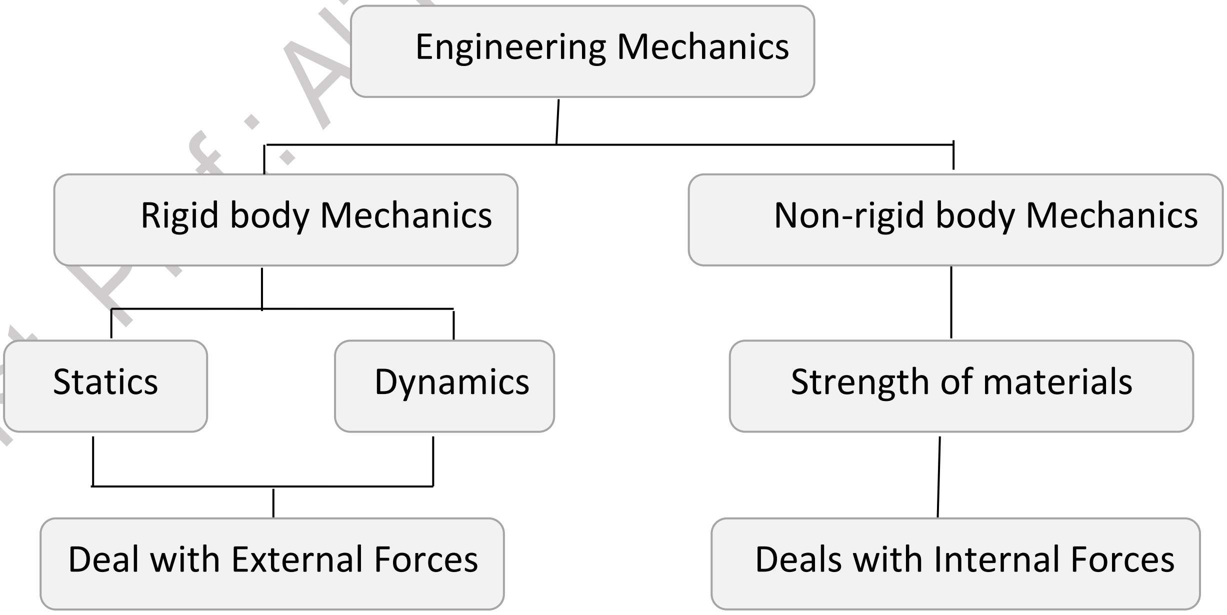
1/1 Introduction:

The word physics comes from the Greek word meaning “nature”. Today physics is treated as the most fundamental branch of science and finds numerous applications of life. Physics deals with matter in relation to energy and the accurate measurement of the same. Thus, physics is inherently a science of measurement. The fundamentals of physics form the basis for the study and the development of engineering and technology.
Measurement consists of the comparison of an unknown quantity with a known fixed quantity. The quantity used as the standard of measurement is called ‘unit’. For example, a vegetable vendor weighs the vegetables in terms of units like kilogram.
Fundamental physical quantities
Fundamental quantities are the quantities which cannot be expressed in terms of any other physical quantity. (e.g.) length, mass and time.

1/2 MECHANICS

Mechanics is the oldest physical science which deals with the effects of forces on objects. The principles of mechanics are central in the fields of vibrations, stability and strength of structures and machines, robotics, rocket and spacecraft design, automatic control, engine performance, fluid flow, electrical machines and apparatus, and molecular, atomic, and subatomic behavior.
The subject of mechanics is logically divided into two parts:
Statics, which concerns the equilibrium of bodies under action of forces, and
Dynamics, which concerns the motion of bodies.
Engineering Mechanics is divided into these two parts, *Statics* and *Dynamics*.



Statics and dynamics are devoted primarily to the study of the external effects upon rigid bodies. In contrast, mechanics of materials deals with the internal effects and deformations that are caused by the applied loads.

1/3 BASIC CONCEPTS

Space: is the geometric region occupied by bodies whose positions are described by linear and angular measurements relative to a coordinate system.

Time: is the measure of the succession of events and is a basic quantity in dynamics.

Mass: is a measure of the inertia of a body, which is its resistance to a change of velocity. The mass of a body affects the gravitational attraction force between it and other bodies.

A particle: is a body of negligible dimensions.

Rigid body: A body is considered rigid when the change in distance between any two of its points is negligible.

Force: is the action of one body on another. A force tends to move a body in the direction of its action. The action of a force is characterized by its magnitude, by the direction of its action, and by its point of application. Thus, force is a vector quantity.

Derived quantities

Quantities that can be expressed in terms of fundamental quantities are called derived quantities. (e.g.) area, volume, density.

Unit

Unit of a physical quantity is defined as the accepted standard used for comparison of given physical quantity. The unit in which the fundamental quantities are measured are called fundamental unit and the units used to measure derived quantities are called derived units.

SI Units

SI unit is the abbreviation for System International de units and is the modern form of metric system finally agreed upon at the eleventh international conference of weights and measures, 1960. This system of units is now being adopted throughout the world and will remain the primary system of units of measurement. SI system possesses features that make it logically superior to any other system. There are seven fundamental units (base units) and two supplementary units.

SI system of units

Physical quantity	Unit	Symbol
Fundamental quantities		
1. Length	meter	m
2.Mass	kilogram	kg
3.Time	second	s
4.Electric current	ampere	A
5.Temperature	kelvin	K
6.Luminous	Intensity candela	cd
7.Amount of substance	mole	mol
Supplementary quantities		
Plane angle	radian	rad

Derived quantities and their units

Sl.No	Physical quantity	Formula	Unit	Symbol
1	Area of the square	side × side	metre^2 or square metre	m^2
2	Volume of the cube	side × side × side	metre^3 or cubic metre	m^3
3	Density	Mass / volume	kilogram metre^{-3}	kgm^{-3}
4	Velocity	Displacement / time	metre second^{-1}	ms^{-1}
5	Acceleration	velocity/ time	metre second^{-2}	ms^{-2}
6	Momentum	mass × velocity	Kilogram metre second^{-1}	$kgms^{-1}$
7	Force	mass × acceleration	newton	N
8	Impulse	force × time	newton second	Ns
9	Work (or) Energy	force × displacement	newton metre or joule	J
10	Power	Work / time	joule second^{-1} or watt	W

Multiplication factor	Prefix	Symbol
1 000 000 000 000 = 10^{12}	Tera	T
1 000 000 000=10^9	Giga	G
1 000 000=10^6	Mega	M
1 000=10^3	Kilo	K
100=10^2	hecto*	h
10=10^1	deca*	da
0.1=10^{-1}	deci	d
0.01=10^{-2}	centi*	c
0.001=10^{-3}	milli	m
0.000 001=10^{-6}	micro	µ
0.000 000 001=10^{-9}	nano	n
0.000 000 000 001=10^{-12}	pico	p
0.000 000 000 000 001=10^{-15}	femto	f
0.000 000 000 000 000 001=10^{-18}	atto	a

*These are not preferred ones. They are used where the other prefixes are inconvenient

1/4 SCALARS AND VECTORS

Scalar quantities are those with which only a magnitude is associated. Examples are time, volume, density, speed, energy, and mass.

Vector quantities, on the other hand, possess direction as well as magnitude, and must obey the parallelogram law of addition as described later in this article. Examples of vector quantities are displacement, velocity, acceleration, force, moment, and momentum. Speed is a scalar. It is the magnitude of velocity.

A ***free vector*** is one whose action is not confined to or associated with a unique line in space. For example, if a body moves without rotation, then the movement or displacement of any point in the body may be taken as a vector, which described by direction and magnitude of the displacement of every point in the body.

A ***sliding vector*** has a unique line of action in space but not a unique point of application.

A ***fixed vector*** is one for which a unique point of application is specified. The action of a force on a deformable or nonrigid body must be specified by a fixed vector at the point of application of the force.

Conventions for Equations and Diagrams

A vector quantity **V** (In handwritten, use a distinguishing mark for each vector quantity, such as an underline, $\underline{V}$, or an arrow over the symbol, $\vec{V}$, to take the place of boldface type in print) is represented by a line segment, Fig. 1/1, having the direction of the vector and having an arrowhead to indicate the sense. The length of the directed line segment represents to some convenient scale the magnitude |**V**| of the vector, which is printed with lightface italic type *V*. For example, we may choose a scale such that an arrow one centimeter long represents a force of twenty newtons.

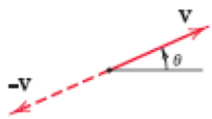


Figure 1/1

Boldface type is used for vector quantities whenever the directional aspect of the vector is a part of its mathematical representation.

The direction of the vector **V** may be measured by an angle ϑ from some known reference direction as shown in Fig. 1/1. The negative of **V** is a vector

-V having the same magnitude as **V** but directed in the sense opposite to **V**, as shown in Fig. 1/1.

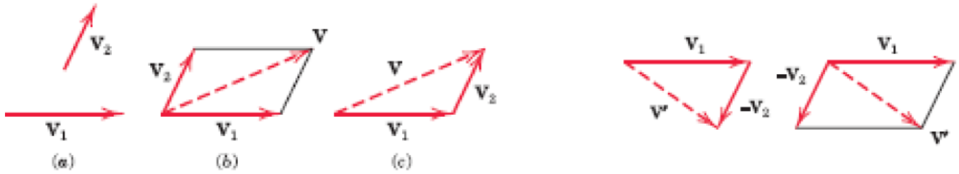


Figure 1/2

Vectors must obey the parallelogram law of combination. This law states that two vectors $\mathbf{V}_1$ and $\mathbf{V}_2$, treated as free vectors, Fig. 1/2*a*, may be replaced by their equivalent vector **V**, which is the diagonal of the parallelogram formed by $\mathbf{V}_1$ and $\mathbf{V}_2$ as its two sides, as shown in Fig. 1/2*b*. This combination is called the ***vector sum***, and is represented by the vector equation

$\mathbf{V} = \mathbf{V}_1 + \mathbf{V}_2 = \mathbf{V}_2 + \mathbf{V}_1$,

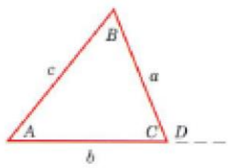
$\mathbf{V'} = \mathbf{V}_1 - \mathbf{V}_2$, where the minus sign denotes *vector subtraction*.

$V \neq V_1 + V_2$,

$V = \sqrt{V_1^2 + V_2^2 - 2\,V_1 V_2 \cos C} = \sqrt{V_1^2 + V_2^2 + 2\,V_1 V_2 \cos D}$

Law of sines

$\frac{a}{b} = \frac{\sin A}{\sin B}$



Law of cosines

$c^2 = a^2 + b^2 - 2ab\cos C$

$c^2 = a^2 + b^2 + 2ab\cos D$

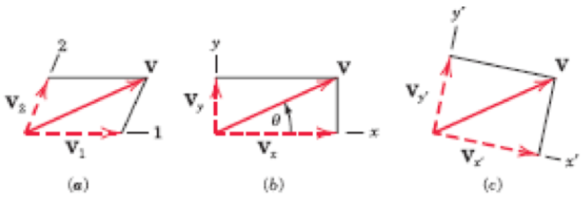


Figure 1/4

Any two or more vectors whose sum equals a certain vector **V** are said to be the ***components*** of that vector. Thus, the vectors $\mathbf{V}_1$ and $\mathbf{V}_2$ in Fig. 1/4*a* are the components of **V** in the directions 1 and 2, respectively.

Vectors $\mathbf{V}x$ and $\mathbf{V}y$ in Fig. 1/4*b* are the *x*- and *y*-components ***(rectangular components)***, respectively, of **V**. Likewise, in Fig. 1/4*c*, $\mathbf{V}x'$ and $\mathbf{V}y'$ are the *x*'- and *y*'-components of **V**. When expressed in rectangular components, the direction of the vector with respect to, say, the *x*-axis is clearly specified by the angle θ, where

$\theta = tan^{-1}\frac{V_y}{V_x}$

A vector **V** may be expressed mathematically by multiplying its magnitude *V* by a vector **n** whose magnitude is one and whose direction coincides with that of **V**. The vector **n** is called a *unit vector*. Thus,

$$\mathbf{V} = V\,\mathbf{n}$$

In many problems, particularly three-dimensional ones, it is convenient to express the rectangular components of **V**, Fig. 1/5, in terms of unit vectors **i**, **j**, and **k**, which are vectors in the *x*-, *y*-, and *z*- directions, respectively, with unit magnitudes. Because the vector **V** is the vector sum of the components in the *x*-, *y*-, and *z*-directions, we can express **V** as follows:

$$\mathbf{V} = V_x\mathbf{i} + V_y\,\mathbf{j} + V_z\mathbf{k}$$

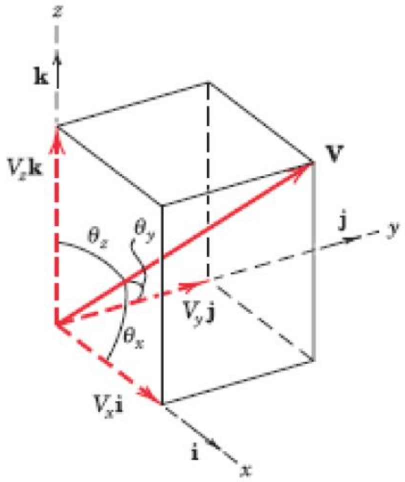


Figure 1/5

We now make use of the ***direction cosines*** ***l, m***, and ***n*** of **V**, which are defined by: $l = \cos\theta_x$, $m = \cos\theta_y$, $n = \cos\theta_z$

Thus, we may write the magnitudes of the components of **V** as

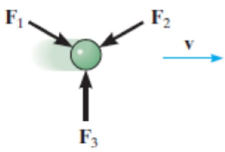
$$V_x = lV \qquad V_y = mV \qquad V_z = nV$$

Where, from the Pythagorean theorem, $V^2 = V_x^2 + V_y^2 + V_z^2$

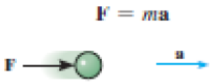
Note that this relation implies that: $l^2 + m^2 + n^2 = 1.$

1/5 NEWTON' S LAWS

Law I. A particle remains at rest or continues to move with *uniform velocity* (in a straight line with a constant speed) if there is no unbalanced force acting on it.



Equilibrium
(a)



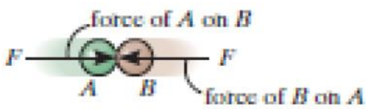
Accelerated motion
(b)

Law II. The acceleration of a particle is proportional to the vector sum of forces acting on it, and is in the direction of this vector sum.

Law III. The forces of action and reaction between interacting bodies are equal in magnitude, opposite in direction, and *collinear* (they lie on the same line).

F = *m***a** (SI unit: newton (N) = kg.m/s^2, U.S. units: pound (Ib)) (1/1)

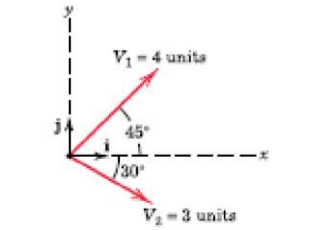
W = *mg* (SI units (m in Kg.), g = 9.806 65 m/s^2) ,
(U.S. units (m in slug.), g = 32.1740 ft/sec^2)



Action – reaction
(c)

Sample Problem

For the vectors **V1** and **V2** shown in the figure,
(*a*) determine the magnitude *S* of their vector sum **S** = $\mathbf{V}_1 + \mathbf{V}_2$
(*b*) determine the angle *α* between **S** and the positive *x*-axis
(*c*) write **S** as a vector in terms of the unit vectors **i** and **j** and then write a unit vector **n** along the vector sum **S**
(*d*) determine the vector difference **D** = $\mathbf{V}_1 - \mathbf{V}_2$



Solution (*a*) We construct to scale the parallelogram shown in Fig. *a* for adding $\mathbf{V}_1$ and $\mathbf{V}_2$. Using the law of cosines, we have

$S^2 = 3^2 + 4^2 - 2(3)(4)\cos 105$

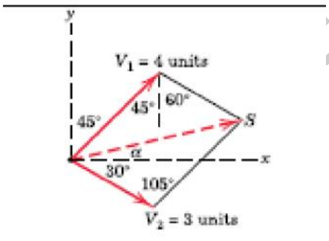
S=5.59 units

(*b*) Using the law of sines for the lower triangle, we have

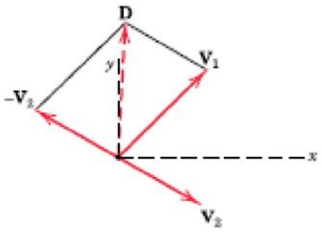
$$\frac{\sin 105}{5.59} = \frac{\sin(\propto +30)}{4}$$

$$(\propto +30) = 43.8$$

$$\propto = 13.76\,°$$



(a)



(b)

(*c*) With knowledge of both *S* and α, we can write the vector **S** as

S= S[i cos α +j sin α]

$= \mathbf{S}[\mathbf{i}\cos 13.76^{o} + \mathbf{j}\sin 13.76^{o}] = 5.43\,\mathbf{i} + 1.328\,\mathbf{j}$ **units** *Ans.*

Then $\mathbf{n} = \frac{S}{S} = \frac{5.43\,i + 1.328\,j}{5.59} = 0.971\,i + 0.238\,j$ *Ans.*

(*d*) The vector difference **D** is

$\mathbf{D} = \mathbf{V}_1 - \mathbf{V}_2 = 4(\mathbf{i}\cos 45° + \mathbf{j}\sin 45°) - 3(\mathbf{i}\cos 30° - \mathbf{j}\sin 30°)$

$= 0.230\mathbf{i} + 4.33\mathbf{j}$ units

The vector **D** is shown in Fig. *b* as $\mathbf{D} = \mathbf{V}_1 + (-\mathbf{V}_2)$.

Helpful Hints

1- You will frequently use the laws of cosines and sines in mechanics. See Art. C/6 of Appendix C for a review of these important geometric principles.

2- A unit vector may always be formed by dividing a vector by its magnitude.
Note that a unit vector is dimensionless

- **Notes:**

It can find the magnitude of |**S**| from analytical solution for *x* and *y* components.

$|S| = \{[V_1(\cos 45) + V_1(\cos 30)]^2 + [V_1(\sin 45) + V_1(\sin 30)]^2\}^{1/2}$

$= \{[S_x]^2 + [S_y]^2\}^{1/2}$

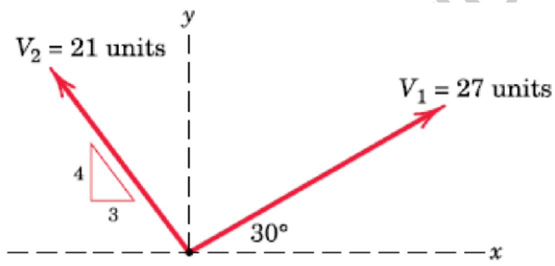
And also find angle (α) from: $\alpha = \tan^{-1}\frac{S_y}{S_x}$

PROBLEMS

1/1 Determine the angles made by the vector **V** = -36**i** +15**j** with the positive x- and y-axes. Write the unit vector **n** in the direction of **V**.

Ans. ϑx = 157.4°, ϑy = 67.4 °, **n**=-0.923**i** + 0.385**j**

1/2 Determine the magnitude of the vector sum **V** = **V**1 + **V**2 and the angle ϑx which **V** makes with the positive x-axis. Complete both graphical and algebraic solutions.



Problem 1/2

1/3 For the given vectors **V**1 and **V**2 of Prob. 1/2, determine the magnitude of the vector difference **V'** = **V**1 + **V**2 and the angle ϑ_x which **V** makes with the positive x-axis. Complete both graphical and algebraic solutions.

Ans. V' = 36.1 units, ϑx = 174.8°

1/4 A force is specified by the vector **F** = 160**i** + 80**j** - 120**k** N. Calculate the angles made by **F** with the positive x-, y-, and z-axes.